Neuron–astrocyte associative memory

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Introduction

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Lagrangians and activation functions
Associative neuron–astrocyte model
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Main idea of the paper

The paper asks whether astrocytes can contribute to associative memory.

- Classical neural network models usually focus on neurons and synapses.
- This paper adds astrocytes as an active computational component.
- The central claim is that neuron–astrocyte interactions can create higher-order memory storage.

$$\text{neurons} + \text{synapses} + \text{astrocyte processes} \implies \text{associative memory}$$

Biological motivation: tripartite synapse

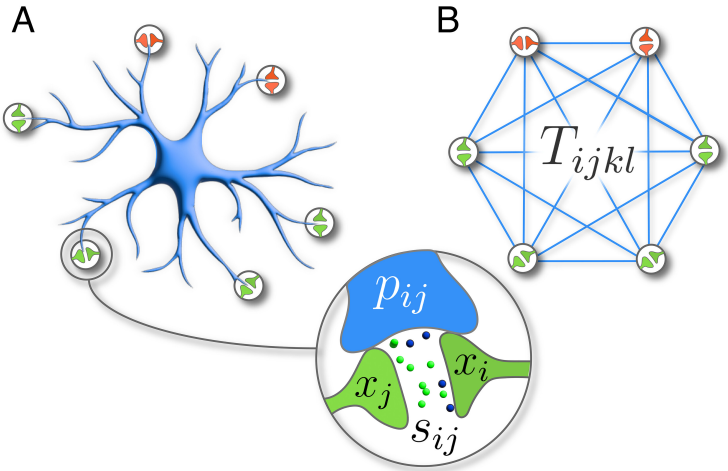
- A normal synapse involves a presynaptic neuron and a postsynaptic neuron.
- Astrocyte processes can wrap around nearby synapses.
- This forms a **tripartite synapse**:

$$\text{pre-neuron} + \text{post-neuron} + \text{astrocyte process}.$$

- The astrocyte process can detect neurotransmitters, produce local Ca^{2+} signals, and release gliotransmitters.

Astrocytes can modulate synaptic transmission instead of only supporting neurons.

astrocyte-neuron



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Intuitive feedback loop

$$\text{synaptic activity} \rightarrow \text{astrocyte } Ca^{2+} \rightarrow \text{gliotransmitter release} \rightarrow$$

$$\text{synaptic efficacy} \rightarrow \text{neural activity}$$

- A local synapse can activate the astrocyte process surrounding it.
- Astrocyte Ca^{2+} activity can influence neurotransmitter release probability or postsynaptic response.
- This creates a feedback loop between neurons, synapses, and astrocytes.

General neuron–astrocyte dynamics

Three types of dynamical variables

The general model contains three coupled variables:

$$x_i = \text{state of neuron } i,$$

$$s_{ij} = \text{synaptic efficacy from neuron } j \text{ to neuron } i,$$

$$p_{ij} = \text{local } \mathrm{Ca}^{2+} \text{ level in the astrocyte process around synapse } j \to i.$$

The model extends a recurrent neural network by adding dynamic synapses and astrocyte calcium dynamics.

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Neuron dynamics

$$\tau_n \dot{x}_i = -\lambda x_i + \sum_{j=1}^{N} g(s_{ij})\phi(x_j) + b_i$$

- $\dot{x}_i$ is the change rate of neuron i.
- $-\lambda x_i$ represents natural leakage or decay.
- $\phi(x_j)$ converts neuron j's state into output activity.
- $g(s_{ij})$ is the effective synaptic weight from neuron j to neuron i.
- b_i is a baseline input.

Compared with a standard RNN, the effective weight is dynamic because it depends on s_{ij}.

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Synaptic dynamics

$$\tau_s \dot{s}_{ij} = -\alpha s_{ij} + f(x_i, x_j, p_{ij}, s_{ij}) + c_{ij}$$

- This equation describes how the synaptic efficacy s_{ij} changes.
- It depends on the postsynaptic neuron x_i and presynaptic neuron x_j.
- It also depends on the local astrocyte Ca^{2+} level p_{ij}.
- Thus, astrocytes can indirectly regulate neuron–neuron connection strength.

Astrocytes enter the model by modulating synaptic efficacy.

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Astrocyte process dynamics

$$\tau_p \dot{p}_{ij} = -\gamma p_{ij} + \sum_{k,l=1}^{N} T_{ijkl}\psi(p_{kl}) + \kappa(s_{ij}) + d_{ij}$$

- p_{ij} is the local Ca^{2+} level in the astrocyte process around synapse $j \to i$.
- $-\gamma p_{ij}$ represents calcium decay.
- $\sum T_{ijkl}\psi(p_{kl})$ represents influence from other astrocyte processes.
- $\kappa(s_{ij})$ represents input from the local synapse.

Astrocyte processes can connect multiple synapses through calcium signaling.

Meaning of the general model

$$x_i \quad \leftrightarrow \quad s_{ij} \quad \leftrightarrow \quad p_{ij}$$

- Neuron activity affects synaptic efficacy.
- Synaptic activity affects local astrocyte Ca^{2+}.
- Astrocyte Ca^{2+} can feed back to synaptic efficacy.
- Different astrocyte processes can also influence one another.

The model is a coupled neuron–synapse–astrocyte dynamical system.

Lagrangians and activation functions

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How activation functions are defined

The paper defines activation functions from Lagrangian functions:

$$\text{activation} = \frac{\partial \mathcal{L}}{\partial z_i}.$$

Here, z can represent a neural, synaptic, or astrocytic dynamical variable.

- The Lagrangian is not the biological mechanism itself.
- It is a mathematical way to define nonlinear activation functions.
- Different Lagrangians lead to different activation functions.

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Two examples of Lagrangians

Collective activation:

$$\mathcal{L}(z) = \log \sum_{i=1}^{N} e^{z_i} \quad \Longrightarrow \quad \frac{\partial \mathcal{L}}{\partial z} = \mathsf{Softmax}(z).$$

Element-wise activation:

$$\mathcal{L}(z) = \sum_{i=1}^{N} Q(z_i) \quad \Longrightarrow \quad \frac{\partial \mathcal{L}}{\partial z} = [q(z_1), \ldots, q(z_N)]^T.$$

These are examples, not the only possible choices.

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From general dynamics to associative memory

- The general model can show complicated behavior, such as oscillations or chaos.
- To study memory, the authors impose special mathematical structure.
- Under symmetry conditions, the model has a global energy function.
- The system evolves toward lower energy states.

$$\frac{dE}{dt} \leq 0$$

Stable fixed points of the energy landscape are interpreted as memories.

Energy-based special case

The paper rewrites the dynamics as a negative gradient of an energy function:

$$\tau_n \dot{x}_i = -\frac{\partial E}{\partial \phi_i} = -\lambda x_i + \sum_{j=1}^{N} g_{ij}\phi_j,$$

$$\tau_s \dot{s}_{ij} = -2\frac{\partial E}{\partial g_{ij}} = -\alpha s_{ij} + \phi_i\phi_j + \psi_{ij},$$

$$\tau_p \dot{p}_{ij} = -2\frac{\partial E}{\partial \psi_{ij}} = -\gamma p_{ij} + \sum_{k,l=1}^{N} T_{ijkl}\psi_{kl} + g_{ij}.$$

The negative signs mean the system moves in the direction of decreasing energy.

Why symmetry is needed

- A global energy function requires interactions to be mathematically consistent.
- This is similar to the symmetry condition in Hopfield networks:

$$g_{ij} = g_{ji}.$$

- For astrocyte processes, the corresponding condition is:

$$T_{ijkl} = T_{klij}.$$

- Intuitively, the influence from process kl to process ij must match the reverse interaction.

Symmetry prevents cyclic dynamics and allows convergence to fixed points.

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Why energy decreases

The energy derivative can be written as:

$$\frac{dE}{dt} = -\left[\tau_n \sum_{i,j} \dot{x}_i \frac{\partial^2 \mathcal{L}^{[n]}}{\partial x_i \partial x_j} \dot{x}_j + \frac{\tau_s}{2} \sum_{i,j,k,l} \dot{s}_{ij} \frac{\partial^2 \mathcal{L}^{[s]}}{\partial s_{ij} \partial s_{kl}} \dot{s}_{kl} + \frac{\tau_p}{2} \sum_{i,j,k,l} \dot{p}_{ij} \frac{\partial^2 \mathcal{L}^{[p]}}{\partial p_{ij} \partial p_{kl}} \dot{p}_{kl}\right].$$

$$\frac{dE}{dt} \leq 0$$

- The Hessian terms come from differentiating the activation functions.
- If the Lagrangians are convex, these quadratic forms are nonnegative.
- The outer negative sign makes the energy non-increasing.

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Effective neuron-only dynamics

At fixed points, the synaptic and astrocyte variables can be eliminated:

$$\psi_{ij} = -\phi_i \phi_j,$$

$$g_{ij} = \sum_{k,l=1}^{N} T_{ijkl} \phi_k \phi_l.$$

Substituting these into the neuron dynamics gives:

$$\tau_n \dot{x}_i = -x_i + \sum_{j,k,l=1}^{N} T_{ijkl} \phi_j \phi_k \phi_l.$$

Astrocytes turn pairwise neuron interactions into higher-order interactions.

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Effective energy

The resulting effective energy is:

$$E^{\text{eff}} = \left[\sum_{i=1}^{N} x_i \phi_i - \mathcal{L}^{[n]}\right] - \frac{1}{4} \sum_{i,j,k,l=1}^{N} T_{ijkl} \phi_i \phi_j \phi_k \phi_l.$$

- The first part is the neuron-layer energy.
- The second part is a fourth-order interaction term.
- This fourth-order term is mediated by astrocyte process interactions.

The astrocyte network creates a Dense Associative Memory-like energy structure.

Storing memories

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Memory patterns

Assume there are K stored memory patterns:

$$\xi^\mu, \qquad \mu = 1, \ldots, K.$$

Each memory pattern is an N-dimensional vector:

$$\xi^\mu = (\xi_1^\mu, \xi_2^\mu, \ldots, \xi_N^\mu).$$

- Each memory is represented as a neural activity pattern.
- The task of associative memory is to store these patterns in network weights.
- A corrupted input should converge back to the closest stored memory.

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Storing memories in the astrocyte tensor

The paper stores memory patterns in the fourth-order tensor:

$$T_{ijkl} = \sum_{\mu=1}^{K} \xi_i^\mu \xi_j^\mu \xi_k^\mu \xi_l^\mu .$$

- Classical Hopfield networks store pairwise relations:

$$W_{ij} = \sum_{\mu} \xi_i^\mu \xi_j^\mu .$$

- This model stores fourth-order relations among neurons.
- The tensor T_{ijkl} represents higher-order interactions mediated by astrocyte processes.

Memories are stored in higher-order neuron group interactions rather than only pairwise synaptic weights.

Dense Associative Memory form

With this choice of T_{ijkl}, the effective energy becomes:

$$E^{\text{eff}} = \left[\sum_{i=1}^{N} x_i \phi_i - \mathcal{L}^{[n]}\right] - \sum_{\mu=1}^{K} F\left(\sum_{i=1}^{N} \xi_i^\mu \phi_i\right),$$

where

$$F(z) = \frac{1}{4} z^4.$$

The effective dynamics are:

$$\tau_n \dot{x}_i = -x_i + \sum_{\mu=1}^{K} \xi_i^\mu F'\left(\sum_{j=1}^{N} \xi_j^\mu \phi_j\right).$$

The network compares the current state with stored memory patterns and moves toward the most similar one.

Memory capacity

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Capacity scaling

The model contains approximately:

$$N \text{ neurons} + N^2 \text{ synapses} + N^2 \text{ astrocyte processes} \sim N^2$$

compute units.

For quartic Dense Associative Memory:

$$K^{\max} \sim N^3.$$

Therefore:

$$\frac{K^{\max}}{\text{number of compute units}} \sim \frac{N^3}{N^2} \sim N.$$

The paper argues that memory capacity per compute unit grows linearly with network size.

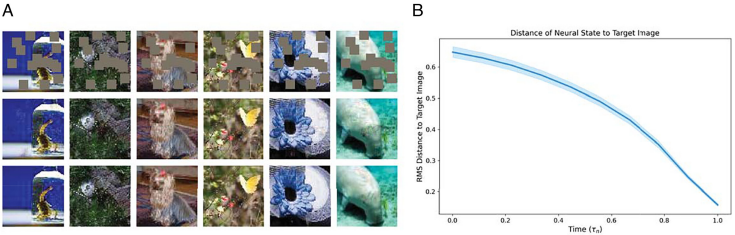

Figure: Top row is the masked input to the network, Middle row is the final state of the network, Bottom row is the ground-truth, unmasked image.
Root-mean-squared distance of the state of the network to the ground-truth image, as function of time. SE was calculated across a batch of 64 images.

Interpretation of the capacity result

- Without astrocytes, memory storage mainly comes from neuron–neuron interactions.
- With astrocytes, many synapses can be connected through astrocyte processes.
- This creates more possible memory attractors in the energy landscape.
- The authors compare this favorably with other biologically plausible Dense Associative Memory implementations.

This is a theoretical capacity argument, not direct biological proof.

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Simulation

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What the simulation tests

- The previous sections build a theoretical associative memory model.
- The simulation checks whether the model actually recovers stored memories.
- The network is given corrupted or noisy inputs.
- The dynamics should converge to the corresponding stored pattern.

$$\text{corrupted input} \rightarrow \text{neuron–astrocyte dynamics} \rightarrow \text{retrieved memory}$$

Simulation result

- The model can retrieve stored patterns from corrupted inputs.
- The energy decreases during the recovery process.
- The final fixed point corresponds to the stored memory.

The simulation supports the claim that the model can implement associative memory.

Discussion

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Main implication

- The paper suggests that memory may not be stored only in neuron–neuron synaptic weights.
- Astrocyte processes may also contribute through calcium signaling and molecular transport.
- This provides a possible biological mechanism for higher-order memory storage.

Astrocytes may be part of the computational machinery of memory.

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Limitations and future directions

- The model is theoretical and idealized.
- The symmetry assumptions are mathematically useful, but real biological circuits may not satisfy them exactly.
- Future experiments could test whether disrupting astrocyte Ca^{2+} signaling impairs memory recall.
- Future models could extend from one astrocyte to larger astrocyte networks.

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Takeaway

The paper turns astrocytes from passive support cells into active components of an associative memory model.

Astrocyte processes may link different synapses, create higher-order interactions, and increase memory storage capacity.